Gravitational Interactions

The equation for the law of universal gravitation is

\[ F = G \frac{m_1 m_2}{d^2} \]

where \( F \) is the attractive force between masses \( m_1 \) and \( m_2 \) separated by distance \( d \). \( G \) is the universal gravitational constant (and relates \( G \) to the masses and distance as the constant \( \pi \) similarly relates the circumference of a circle to its diameter). By substituting changes in any of the variables into this equation, we can predict how the others change. For example, we can see how the force changes if we know how either or both of the masses change, or how the distance between their centers changes.

Suppose, for example, that one of the masses somehow is doubled. Then substituting \( 2m_1 \) for \( m_1 \) in the equation gives

\[ F_{\text{new}} = G \frac{2m_1 m_2}{d^2} = 2G \frac{m_1 m_2}{d^2} = 2F_{\text{old}} \]

So we see the force doubles also. Or suppose instead that the distance of separation is doubled. Then substituting \( 2d \) for \( d \) in the equation gives

\[ F_{\text{new}} = G \frac{m_1 m_2}{(2d)^2} = G \frac{m_1 m_2}{4d^2} = \frac{1}{4} \frac{G m_1 m_2}{d^2} = \frac{1}{4} F_{\text{old}} \]

And we see the force is only \( \frac{1}{4} \) as much.

Use this method to solve the following problems. Write the equation and make the appropriate substitutions.

1. If both masses are doubled, what happens to the force?

\[ F_{\text{new}} = G \frac{2m_1 2m_2}{d^2} = 4G \frac{m_1 m_2}{d^2} = 4F_{\text{old}} \]

2. If the masses are not changed, but the distance of separation is reduced to half the original distance, what happens to the force?

\[ F_{\text{new}} = G \frac{m_1 m_2}{(d/2)^2} = G \frac{m_1 m_2}{d^2} = 4G \frac{m_1 m_2}{d^2} = 4F_{\text{old}} \]
3. If the masses are not changed, but the distance of separation is reduced to one fourth the original distance, what happens to the force?

\[ F_{\text{new}} = G \frac{m_1 m_2}{(d/4)^2} = G \frac{m_1 m_2}{16d^2} = 16 G \frac{m_1 m_2}{d^2} = 16 F_{\text{old}} \]

4. If both masses are doubled, and the distance of separation is doubled, show what happens to the force.

\[ F_{\text{new}} = G \frac{2m_1 2m_2}{(2d)^2} = \frac{4}{4} G \frac{m_1 m_2}{d^2} = F_{\text{old}}; \text{ NO CHANGE} \]

5. If one of the masses is doubled, the other remains unchanged, and the distance of separation is tripled, show what happens to the force.

\[ F_{\text{new}} = G \frac{2m_1 m_2}{(3d)^2} = \frac{2}{9} G \frac{m_1 m_2}{d^2} = \frac{2}{9} F_{\text{old}} \]

6. Consider a pair of binary stars that pull on each other with a certain force. Would the force be larger or smaller if the mass of each star were three times as great when their distance apart is three times as far? Show what the new force will be compared to the first one.

\[ F_{\text{new}} = G \frac{3m_1 3m_2}{(3d)^2} = \frac{9}{9} G \frac{m_1 m_2}{d^2} = F_{\text{old}}; \text{ NO CHANGE} \]